

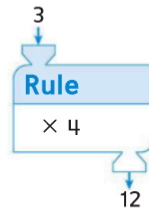
## Function Machines and "What's My Rule?"

A **function machine** is an imaginary machine. The machine is given a rule for changing numbers. You drop a number into the machine. The machine uses the rule to change the number. The changed number comes out of the machine.

Here is a picture of a function machine.

The machine has been given the rule  $\times 4$ .

The machine will multiply any number that is put into it by 4.



### Example

If you drop 3 into the function machine above, it will multiply  $3 \times 4$ . The number 12 will come out.

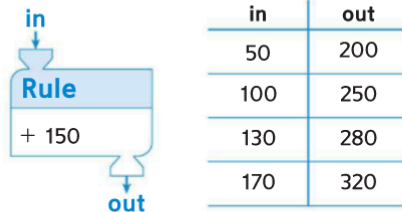
If you drop 1 into the machine, it will multiply  $1 \times 4$ . The number 4 will come out.

If you drop 0 into the machine, it will multiply  $0 \times 4$ . The number 0 will come out.

You can use a table of *in* and *out* numbers to keep track of the way a function machine changes numbers.

Write the numbers that are put into the machine in the *in* column.

Write the numbers that come out of the machine in the *out* column.



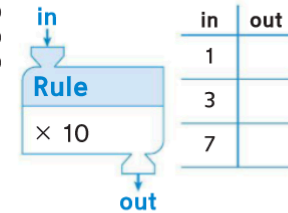
### Example

The rule is  $\times 10$ . You know the numbers that are put into the machine. Find the numbers that come out of the machine.

If 1 is put in, then 10 comes out.  $1 \times 10 = 10$

If 3 is put in, then 30 comes out.  $3 \times 10 = 30$

If 7 is put in, then 70 comes out.  $7 \times 10 = 70$



### Example

The rule is  $\div 2$ . You know the numbers that come out of the machine. Find the numbers that were put into the machine.

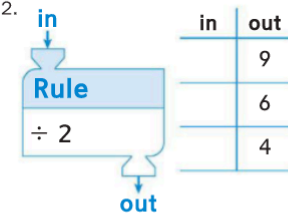
The machine divides any number put into it by 2.

The number that comes out is always half the number that was put in.

If 9 comes out, then 18 was put in.

If 6 comes out, then 12 was put in.

If 4 comes out, then 8 was put in.

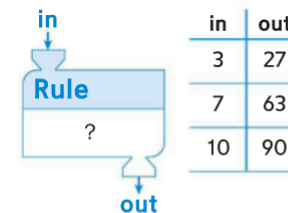


### Example

The rule is not known. Use the table to find a rule.

Each number in the *out* column is 9 times the number in the *in* column.

A rule can be Multiply by 9, or  $\times 9$ .



## Partial-Sums Addition

You can use different methods to add. When you use **partial-sums addition**, you can think of each addend in expanded form. Then you can add the 100s, add the 10s, and add the 1s. Finally, add the partial sums you found.

Use an estimate to check whether your answer is reasonable.

**Note** If you can add the 100s, 10s, and 1s in your head and if you can estimate using mental math, then you don't need to write the steps shown in green.

### Example

$248 + 187 = ?$

Estimate: 248 is close to 250, and 187 is close to 200.

$250 + 200 = 450$

The exact sum should be close to 450.

Use partial-sums addition to add:

Think:  $248 = 200 + 40 + 8$

$187 = 100 + 80 + 7$

Add the 100s.  $200 + 100 \rightarrow$

Add the 10s.  $40 + 80 \rightarrow$

Add the 1s.  $8 + 7 \rightarrow$

Add the partial sums.

	2	4	8
+	1	8	7
	3	0	0
	1	2	0
	8	7	5
	4	3	5

$248 + 187 = 435$

435 is close to the estimate of 450, so 435 makes sense.

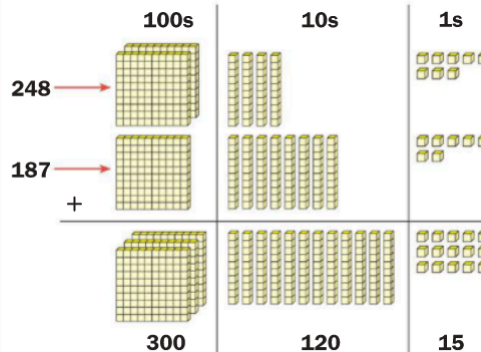
Numbers with 4 or more digits can be added in the same way.

You can use base-10 blocks to show how partial-sums addition works.

### Example

Use base-10 blocks to add  $248 + 187$ .

Adding the blocks in each column is adding the 100s, 10s, and 1s. Then find the total.



Find the total.  $300 + 120 + 15 = 435$

$248 + 187 = 435$

A cube is worth 1.  
A long is worth 10.  
A flat is worth 100.

How are partial-sums addition and addition with base-10 blocks the same? How are they different?

## Column Addition

You can use **column addition** to find sums with paper and pencil.

To use column addition:

- Draw lines to separate the 1s, 10s, and 100s places.
- Add each place-value column. Write each sum in its column.
- If the sum of any column is a 2-digit number, make a trade with the column to the left.

You can use an estimate to check whether your answer is reasonable.

### Example

$$248 + 187 = ?$$

Estimate: 248 is close to 250, and 187 is close to 200.

$$250 + 200 = 450$$

The exact sum should be close to 450.

Add the numbers in each column.

Trade 10 ones for 1 ten.

Move 1 ten to the tens column.

Trade 10 tens for 1 hundred.

Move the 1 hundred to the hundreds column.

$$248 + 187 = \mathbf{435}$$

435 is a reasonable answer because it is close to the estimate of 450.

2	4	8
+	1	87
3	12	15
3	13	5
4	3	5

### Check Your Understanding

Add. Estimate to check whether your answers are reasonable.

1.  $37 + 96$     2.  $159 + 227$     3.  $487 + 361$     4.  $153 + 88$

Check your answers in the Answer Key.

## Expand-and-Trade Subtraction

One method you can use to subtract is called **expand-and-trade subtraction**.

### Example

$$932 - 356 = ?$$

Estimate: 932 is close to 950, and 356 is close to 350.  $950 - 350 = 600$

Use expanded form to write the problem so you can see the hundreds, tens, and ones.

$$\begin{array}{r} 932 \rightarrow 900 + 30 + 2 \\ - 356 \rightarrow 300 + 50 + 6 \\ \hline \end{array}$$

Look at the hundreds. Since  $900 > 300$ , you can use this method to subtract.

Look at the tens. Since  $30 < 50$ , make a trade.

$$\begin{array}{r} 932 \rightarrow 900 + 30 + 2 \\ - 356 \rightarrow 300 + 50 + 6 \\ \hline \end{array}$$

Trade 1 hundred for 10 tens.

Look at the ones. Since  $2 < 6$ , make a trade.

$$\begin{array}{r} 932 \rightarrow 900 + 30 + 2 \\ - 356 \rightarrow 300 + 50 + 6 \\ \hline \end{array}$$

Trade 1 ten for 10 ones.

Subtract the hundreds, tens, and ones.  
 $500 + 70 + 6 = 576$ ,  
 so 576 is the answer.

$$\begin{array}{r} 932 \rightarrow 900 + 30 + 2 \\ - 356 \rightarrow 300 + 50 + 6 \\ \hline 500 + 70 + 6 = 576 \end{array}$$

$$932 - 356 = \mathbf{576}$$

576 is close to the estimate of 600, so 576 makes sense.

### Check Your Understanding

Use expand-and-trade subtraction to find the difference.  
 Estimate to check whether your answers are reasonable.

1.  $93 - 46$     2.  $835 - 451$     3.  $520 - 148$

Check your answers in the Answer Key.

## Trade-First Subtraction

Another method you can use to subtract is called **trade-first subtraction**.

To use trade-first subtraction, look at the digits in each place:

- If a digit in the top number is greater than or equal to the digit below it, you do not need to make a trade.
- If any digit in the top number is less than the digit below it, make a trade with the digit to the left.
- After making all necessary trades, subtract in each column.

Use an estimate to decide whether your answer makes sense.

### Example

$$352 - 164 = ?$$

Estimate: 352 is close to 350. You can round 164 to 160.  
An estimate is  $350 - 160 = 190$ .

100s	10s	1s
3	5	2
- 1	6	4
<hr/>		

Look at the 100s place. Since  $300 > 100$ , there is no trade to make.

100s	10s	1s
2	15	2
- 1	6	4
<hr/>		

Look at the 10s place. Since  $50 < 60$ , you need to make a trade with the column to the left.

100s	10s	1s
2	<sup>14</sup> <del>5</del>	<sup>12</sup> <del>2</del>
- 1	6	4
<hr/>		
1	8	8

Look at the 1s place. Since  $2 < 4$ , you need to make a trade with the column to the left. Now subtract in each column in any order.

$$352 - 164 = \mathbf{188}$$

The answer 188 makes sense because it is close to the estimate of 190.

**Note** If you can keep track of the places in your head, you don't need to draw lines between columns or label the columns. This problem in the example would look like this:

$$\begin{array}{r} 14 \\ 2 \cancel{5} \cancel{2} \\ - 1 \ 6 \ 4 \\ \hline 1 \ 8 \ 8 \end{array}$$

Base-10 blocks are useful for solving problems. If you don't have blocks, you can draw pictures instead.

### Base-10 Blocks Shorthand Pictures

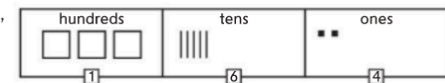
= .	=	= □
cube	long	flat

### Example

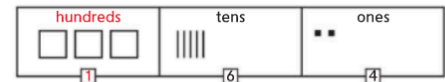
$$352 - 164 = ?$$

Use pictures of base-10 blocks to model the larger number, 352.

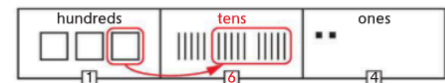
Write the number to be subtracted, 164, beneath the block pictures.



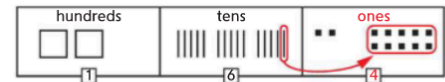
*Think:* Can I remove 1 flat from 3 flats? Yes.



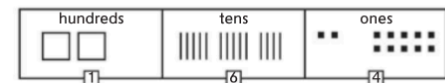
*Think:* Can I remove 6 longs from 5 longs? No. Trade 1 flat for 10 longs.



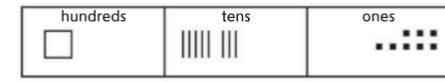
*Think:* Can I remove 4 cubes from 2 cubes? No. Trade 1 long for 10 cubes.



After all of the trading, the blocks look like this:



Now subtract in each column. The remaining blocks show 1 flat, 8 longs, and 8 cubes, which represent 188.



The difference is 188. So,  $352 - 164 = \mathbf{188}$ .

## Counting-Up Subtraction

You can find the difference between two numbers by counting up from the smaller number to the larger number. Subtracting this way is called **counting-up subtraction**. There are many ways to count up. It helps to think of counting to easier numbers, such as numbers that end in zero, or counting by 10s and 100s. One way is to start by counting up to the nearest multiple of 10, then continue counting by 10s and 100s.

You can keep track of your thinking by showing jumps on an open number line.

### Example

$$325 - 88 = ?$$

Estimate:  $325 - 88$

$$\begin{array}{r} \downarrow \quad \downarrow \\ 300 - 100 = 200 \end{array}$$

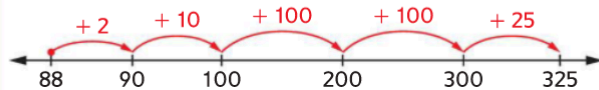
The difference should be close to 200.

Draw a line. Mark and label point 88.



Think: How can I get from 88 to 325?

Start at 88. Count up 2 to get to 90. Count up 10 to 100. Count up by hundreds to get to 300. Count up 25 more to get to 325.



Add your jumps. You counted up  $2 + 10 + 100 + 100 + 25 = 237$ .

$$325 - 88 = \mathbf{237}$$

The answer 237 makes sense because it is close to the estimate of 200.

Another way to keep track of your thinking as you count up is to write number sentences.

### Example

$$325 - 88 = ?$$

Start with the smaller number, 88, and count up to 325. Circle each amount that you count up.

**One way:**

$$\begin{array}{r} 88 \\ + \quad \textcircled{2} \\ \hline 90 \end{array} \quad \text{Count up to the nearest 10.}$$

$$\begin{array}{r} 90 \\ + \quad \textcircled{10} \\ \hline 100 \end{array} \quad \text{Count up to the nearest 100.}$$

$$\begin{array}{r} 100 \\ + \quad \textcircled{200} \\ \hline 300 \end{array} \quad \text{Count up to the largest possible hundred.}$$

$$\begin{array}{r} 300 \\ + \quad \textcircled{25} \\ \hline 325 \end{array} \quad \text{Count up to the larger number.}$$

**Another way:**

$$\begin{array}{r} 88 + \quad \textcircled{2} = 90 \\ 90 + \quad \textcircled{10} = 100 \\ 100 + \quad \textcircled{200} = 300 \\ 300 + \quad \textcircled{25} = 325 \end{array}$$

$$2 + 10 + 200 + 25 = 237$$

$$325 - 88 = \mathbf{237}$$

Then add the numbers you circled:

$$2 + 10 + 200 + 25 = 237$$

You counted up by 237.

$$325 - 88 = \mathbf{237}$$

### Check Your Understanding

Use counting-up subtraction to subtract. Estimate to check whether your answers are reasonable.

1.  $90 - 33$
2.  $242 - 70$
3.  $742 - 387$
4.  $360 - 179$

Check your answers in the Answer Key.

## Tally Charts

There are different ways you can collect information about something:

- count
- measure
- ask questions
- look at something and describe what you see

The information you collect is called **data**. Sometimes you can use a **tally chart** to record and organize data.

### Example

Mr. Davis asked each student to name his or her favorite drink. He recorded the students' choices in the tally chart below.

#### Favorite Drinks

Drink	Tallies
Milk	
Chocolate milk	
Soft drink	
Apple juice	
Tomato juice	
Water	



Milk (5 votes) is more popular than chocolate milk (3 votes).



Soft drink is the most popular choice (11 votes).



Tomato juice is the least popular choice (1 vote).

### Check Your Understanding

- Use the Favorite Drinks tally chart on this page to answer the questions.
  - Which drinks are less popular than apple juice?
  - How many more children chose milk than chocolate milk?
  - How many fewer children voted for apple juice than soft drink?
  - What is the total number of children who voted?

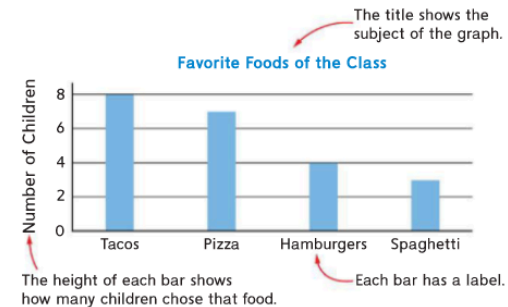
Check your answers in the Answer Key.

## Bar Graphs

A **bar graph** is a drawing that uses bars to represent data. Bar graphs can help you answer questions about the data. The example below is a **scaled bar graph**. The scale shows intervals of 2.

### Example

The bar graph below shows how many children in a class chose certain foods as their favorites.



How many children chose pizza?

The bar for pizza ends halfway between the line for 6 and the line for 8, so 7 children chose pizza as their favorite food.

How many more children chose tacos than spaghetti?

Eight children chose tacos as their favorite food, but only 3 children chose spaghetti. Five more children chose tacos than spaghetti.

Often, you choose the scale for a bar graph based on the data and the amount of available space for the graph. If the numbers in your data set are spread out, you will want to use larger intervals to create your bar graph.